Time-Invariant Structure of Nonstationary Atmospheric Turbulence

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Introduction

NONSTATIONARY turbulence $w(t)$ is defined as that whose correlation function $C(\ldots)$ varies with time, in particular

$$\langle w(t_1) w(t_2) \rangle = C(t,\tau)$$

where $t=(t_1+t_2)/2$ and $\tau=t_2-t_1$. In airplane preliminary design analyses, this time variability introduces certain analytical difficulties, since in order to incorporate turbulence response characteristics into such analyses the design engineer must make some rather ad hoc assumptions about the nature of the turbulence the aircraft is expected to encounter in its day-to-day flight envelope. Indeed, for any practical engineering results to emerge, the design engineer must impose a *particular* time variation on $C(t,\tau)$. The τ dependence is usually well known and can be rigorously modeled using the Dryden formulation, viz., exp $(-|\tau|/\Lambda)$ where $\Lambda\sim$ *integral scale*. A typical time variation imposed by design engineers is the "one-minus-cosine" gust variation. Such variations can often lead to fatigue life problems through nonconservative failure criteria for the aircraft structure, especially if the aircraft ocassionally encounters turbulence whose time variation is something other than the "one-minus-cosine" for which it was designed; such occurrences are not *at all uncommon*.

In this Note, it is established that nonstationary atmospheric turbulence has a distinct well-defined *time-invariant* statistical structure. This structure is duly posed in terms of the aircraft response parameters. To the aeronautical design engineer, this time-invariant characteristic suggests that there is a mode in which the effects of nonstationary turbulence can be incorporated into preliminary design analyses in a very unified and cogent fashion. To be sure, the present results remove the need for making arbitrary decisions and/or assumptions about the time dependence of the statistical structure of nonstationary turbulence; simultaneously, they also show how the effects of virtually *all* types of nonstationary turbulent behavior can be incorporated into a *single* response statistic.

Discussion

The statistical model of turbulence now in practical use is the "uniformly modulated" model,[1-6] in which $C(t,\tau) \simeq \sigma^2(t)R(\tau)$. In this model $\sigma(\cdot)$ is the turbulence intensity and is a function of time, while $R(\tau)$ is a time-invariant correlation function such that $R(0)\equiv 1$; this model agrees well with experimentally obtained turbulence data and, accordingly, produces a time-independent integral scale $\Lambda=\int_0^\infty R(\tau)\mathrm{d}\tau$.

Received March 30, 1985; revision received May 2, 1985; presented as Paper 85-1834 at the AIAA 12th Atmospheric Flight Mechanics Conference, Snowmass, CO, Aug. 19-21, 1985.

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In this model, the function $C(\ldots)$ varies with time, but its geometric shape (i.e., when plotted $C(\ldots)$ vs τ) at any time is *similar* to its geometric shape at any other time, becoming a sort of self-preserving or self-similar nonstationary correlation function (see Fig. 1). The only condition pertinent to the "physics" of the turbulence necessary for this phenomenon to exist is that the intensity vary "slowly" with time, slowly enough that there are no convolution effects between the time changes in $\sigma(\ldots)$ and the integral scale function $R(\tau)$. The time evolution of such a correlation function can be modeled in a unified mode by what is called in geometric transformation theory *affine transformations.* Affine transformations have recently found their way into the analysis of "engineering-type" problems in such fields as flutter analysis,[7,8] analysis of shaky structures,[9,10] and the decay of isotropic turbulence.[11] Their applicability to the analysis of stochastic phenomena in general is discussed in Refs. 12-15.

In practical terminology, affine transformations are coordinate transformations that automatically incorporate the effect of *uniform* changes in coordinate axis scale(s). In aircraft turbulence response problems, changes in the coordinate axis scale manifest themselves through changes in the magnitude of the turbulence intensity, i.e., increases or decreases in intensity can be modeled simply by changing the ordinate axis scale when plotting $C(\ldots)$ vs τ (see Fig. 2).

The intriguing characteristic of affine transformations is that they do not preserve distance between two points.[16] Rather, they preserve the ratio of two *collinear* distances. The reason that they preserve this ratio is that affine transformations characteristically "distort," i.e., they either "shrink" or "stretch," all distances by a factor dependent upon, among other things, the particular transformation itself and therefore an invariant is found to exist only in said ratio. In the analysis of turbulence, this invariance means that the triple-correlation,

$$T(\ldots) = \langle w(t_1) w(t_2) w(t_3) \rangle$$

when modeled in terms of the ratio

$$r = \frac{(t_2-t_1)}{(t_3-t_1)} = \frac{\tau_1}{\tau_2}$$

is time independent, even though the *turbulence itself is fully* nonstationary.[15] In gust response parameter computations, viz., the Duhamel superposition integral,

$$q(t) = \int h(\tau) w(t-\tau)\mathrm{d}\tau \tag{1}$$

suggests that $\langle q^3 \rangle$ is also time independent and, therefore, that this is the statistic whose computation is required in order to determine the cumulative fatigue effects of nonstationary turbulence on aircraft response. In terms of the spectra, this statistic expresses as

$$\langle q^3 \rangle = \left(\frac{1}{2\pi}\right)^3 \int H^*(\omega_1+\omega_2) H(\omega_1) H(\omega_2) B(\omega_1,\omega_2)\mathrm{d}\omega_1 \mathrm{d}\omega_2 \tag{2}$$

where $H(\ldots)$ is Sears' function and $B(\omega_1,\omega_2)$ the bispectrum of the turbulence.[17-20] The bispectrum is not the same as the classical power spectrum and is instead the Fourier transform of the double time-lagged correlation function, i.e.,

$$B(\omega_1,\omega_2) = \int T(\tau_1,\tau_2)\exp\{-i(\omega_1\tau_1+\omega_2\tau_2)\}\mathrm{d}\tau_1\mathrm{d}\tau_2 \tag{3}$$

It is more appropriately denoted as energy-transfer spectrum.

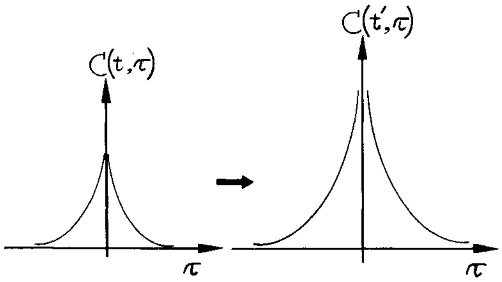

Fig. 1 "Representative" time evolution for "uniformly modulated" turbulence.

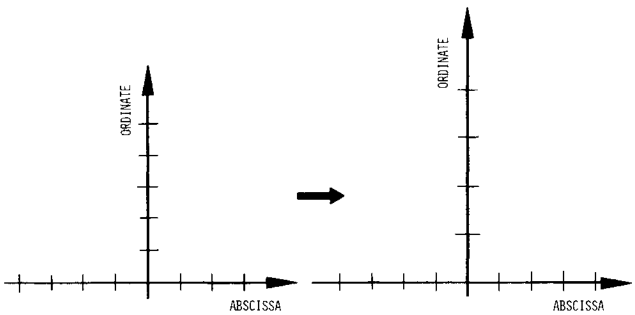

Fig. 2 Ordinate axis scale change effectively capturing the "uniformly modulated" structure.

Conclusions

Numerical results for this analysis are not available at this time since little is known about the functional form of $B(\omega_1,\omega_2)$ for atmospheric turbulence. This Note serves partly to introduce formally the applicability and advantages of bispectral concepts—concepts already employed in related fields[21-28] to aircraft response analyses. One unique feature of the bispectrum of clear importance in atmospheric turbulence is that it also *automatically* incorporates into its functional form the effects of the non-Gaussian statistical structure of the turbulence; the bispectrum for Gaussian turbulence is *identically* zero.

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Subcritical Damping Ratios of a Two-Dimensional Airfoil in Transonic Flow

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Introduction

THE *U-g* method gives critical flutter speeds that are in agreement with the traditional British approach with lined-up frequency parameters, but overestimates the relative damping ratio at other speeds.[1] However, useful values of

Received Jan. 1, 1985.

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